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SOME EXAMPLES OF COGNITIVE TASK ANALYSIS  
WITH INSTRUCTIONAL IMPLICATIONS

James G. Greeno

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Analyses are described of knowledge structures used to understand and solve problems in high school geometry and in primary-grade arithmetic word problems. Analysis of geometry problem solving has clarified the nature of strategic knowledge needed by students and raises the question whether more explicit training in strategies would be beneficial in school instruction. Analysis of semantic knowledge needed to understand word problems raises questions about relationships between students' learning of computational procedures and their understanding of general types of quantitative relationships.

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## SOME EXAMPLES OF COGNITIVE TASK ANALYSIS

### WITH INSTRUCTIONAL IMPLICATIONS

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University of Pittsburgh

As concepts and methods for the analysis of complex cognitive performance have developed, it has been increasingly attractive to think about their potential use in analyzing tasks that are used in instruction. The idea of applying concepts and methods of cognitive psychology to the analysis of instructional tasks is certainly not new; efforts of such early investigators as Dewey, Judd, and Thorndike come to mind, as well as more recent contributions by Atkinson, Gagné, Glaser, Resnick, Skinner, and Suppes, to name a few. However, recent developments seem to have added a new dimension to the potential use of ideas from psychology and other cognitive sciences in the analysis and design of instruction. At least that seemed the case to me when I wrote a chapter entitled "Cognitive Objectives of Instruction," in 1974 (Greeno, 1976a). The organizers of this conference requested that I prepare a paper on that same topic. Perhaps it will be useful in this context if I present a brief progress report of work that I have been engaged in during the meantime. Much of this work is still in very early stages, and I apologize that this presentation is still more a research program than a set of results. However, some of the potential research that I sketched in 1974 has become actual research, and it may be useful to report the directions in which those ideas have developed during the short period since publication of that earlier article.



In my earlier paper, I discussed three kinds of instructional tasks: performing calculations in arithmetic, proving theorems and solving other problems in geometry, and understanding concepts in science. I did not intend to suggest then, nor do I now, that these topics exhaust the instructional domains in which cognitive science will contribute to instructional practice. For example, my short list did not include the analysis of reading skill, which probably is the domain in which the most has been accomplished in relating cognitive science and instruction. However, the three tasks that I discussed represent three important theoretical foci, and my research has progressed in ways that are relevant to those three kinds of tasks. The analysis of calculating skills uses concepts in the theory of cognitive procedures. The analysis of knowledge acquired in geometry uses concepts in the theory of problem solving. And the analysis of understanding scientific concepts uses concepts in the theory of semantic schemata used in the process of understanding language. The work that I will discuss in this paper has involved analyses of geometry problem solving and arithmetic. Thus far, our studies of geometry have fit rather well into the research domain of problem solving. However, in our study of arithmetic we have become concerned with processes of understanding and semantic schemata, as well as with procedural knowledge involved in computational skill.

#### Problem Solving in Geometry

When the cognitive processes involved in an instructional task have been analyzed, the results can be viewed as a hypothesis about the knowledge that students acquire when they successfully learn the material given in instruction. The knowledge required for problem

solving in geometry has been represented in a computer simulation model that I have given the name Perdix. The major source of empirical data used in developing Perdix was a set of thinking-aloud protocols that I obtained from a group of six ninth-grade students during a year in which they were studying geometry in a course. I interviewed the students individually approximately once each week throughout the year. At each session, the student solved a few problems, thinking-aloud during the process. The protocols were recorded on audiotape and the transcriptions are accompanied by diagrams that the students drew during problem solving. In developing Perdix, I have included procedures and structures of knowledge that enable the model to solve the problems that these students were able to solve, in the same general ways that the students solved the problems.

The form of Perdix is a production system, which means that each component of its knowledge is a pair consisting of a condition and an action that is performed if a test of the condition is performed and it is found to be true. The productions that constitute Perdix' knowledge about geometry are in three groups, and these three groups of productions can be considered as three domains of knowledge required for students to solve the problems they are given in their study of geometry.

The three domains of knowledge required for geometry problem solving are the following:

1. Propositions used in making inferences.
2. Perceptual concepts used in recognizing patterns.
3. Strategic principles used in setting goals and planning.



The propositions needed in geometry problem solving are the familiar statements about geometric relations, such as, "Corresponding angles formed by parallel lines and a transversal are congruent"; or, "If a triangle has two sides and the included angle congruent to two sides and the included angle of another triangle, the triangles are congruent"; or, "If two angles are congruent, they have equal measure." Inferences based on this kind of proposition constitute the main steps in geometry problem solving. Geometry problems require students to show relationships between objects, for example, "Prove that angle A and angle B are congruent";) or to find the measure of an object, such as the size of angle or the length of a line segment. Information is given in the problem in the form of segments or angles that are congruent, lines that are parallel, the measures of some angles or segments, and so on. Each step in solving the problem consists of an inference in which some new relation or the measure of some additional component is deduced from information that was given or that has previously been inferred. The problem is solved when this chain of inferences reaches the relation or measure that is the goal of the problem. Each of the inferential steps is based on one of the if-then propositions that the student knows. The antecedent condition of the proposition is found in the given information or the diagram, and the consequent relation is added to the problem situation.

The perceptual concepts needed for geometry problem solving include the patterns that are mentioned in the antecedents of many propositions. For example, the proposition, "Corresponding angles formed by parallel lines and a transversal are congruent" mentions a pattern, corresponding angles. To use this proposition as a basis for inferring that angles



are congruent, a student must look at a diagram and determine that the angles are in the correct positions relative to a pair of parallel lines and a transversal to be called corresponding angles.

The strategic knowledge that is needed in geometry includes knowledge of general plans that lead to the various kinds of goals that occur in geometry problems. For example, when solution of a problem requires showing that two angles are congruent, three alternative approaches are available. One approach is to prove that triangles containing the angles are congruent. A second approach is to use relations between angles that are based on parallel lines, such as corresponding angles or alternate interior angles. A third approach is to use relationships between angles whose vertices are at the same point, such as vertical angles, or angles that are formed by the bisection of another angle.

The design of the planning process in Perdix is similar to the one developed by Sacerdoti (1975) in his program NOAH (for Nets of Action Hierarchies). As with NOAH, Perdix has knowledge of some general actions that it can perform. Knowledge about each general action includes the consequences of the action and prerequisite conditions that are required for the action to be performed. Perdix selects a plan for its current goal by checking the general action that have consequences that achieve the goal. If the prerequisite conditions for one of the actions are present in the situation, Perdix adopts the plan of achieving the goal using that action. Then Perdix proceeds to try to execute the plan, using procedures that are also stored as part of the knowledge that Perdix has about the general action. These procedures can include the setting of further goals, which may require selection of plans for

their achievement, leading to a hierarchy of plans and goals for the solution of the problem.

Most of the features of the model for geometry problem solving have been developed by applying standard concepts in the recent literature on problem solving in psychology and artificial intelligence. There have been some interesting new developments required to simulate problem solving in this domain, which are discussed in other places (Greeno, 1976b, 1977, 1978). However, the main results have been obtained by examining the nature of the geometry task environment in some detail, studying the performance of subjects who are successful in performing the tasks that are used as a criterion of learning in that domain, and using concepts and methods that have been worked out in the general theory of problem solving to develop a theory about the knowledge structures and cognitive processes required for successful performance in the domain.

The result of this theoretical analysis can be considered as a model of the outcome of successful instruction for those aspects of the course that have been included in the analysis thus far. It has the advantage over purely rational task analysis that it is generally consistent with performance of human learners who did succeed in learning how to accomplish the criterion tasks. On the other hand, it does not characterize all the students who were in the course; some of them did not succeed in acquiring the necessary knowledge, and I do not have a model for their unsuccessful performance. Furthermore, to provide a really strong guide for instructional practice, we need to develop models of the process of acquisition in addition to models of the knowledge that is acquired.



On the other hand, a clear representation of the outcome of successful instruction probably can be useful. In the case of this geometry model, some interesting issues appear when the characteristics of the model are considered in relation to the content of the geometry curriculum as it is represented in texts for the course.

The theoretical analysis of geometry problem solving led to the conclusion that three main components of knowledge are required for a student to successfully accomplish the criterion tasks used in the domain. These are propositions for inference, perceptual concepts for pattern recognition, and strategic knowledge for planning and setting goals. Of these three, the first two are included explicitly in the instructional materials used in teaching. There is explicit presentation of the propositions that are used as the basis of inferences.

When a new proposition is introduced, it is always explained carefully, and often a proof of the proposition is given. There is also explicit presentation of the perceptual concepts that are needed for pattern recognition. These are usually presented in diagrams, with exercises that emphasize the relevant features needed to identify instances of the concepts.

However, the components that I have been calling strategic knowledge are not represented explicitly in the instructional materials of geometry. The knowledge that is needed for planning and setting goals can be given an explicit characterization; indeed, it has such a characterization in the model that I have been describing. However, most references to that knowledge in texts that I have examined are relatively indirect, and my impression is that most teachers do not explicitly identify principles of strategy when they teach their students.



One interesting question is the following: If the instructional materials of a course do not include an important part of the knowledge needed to perform criterion tasks, how do students acquire that knowledge? We know that many students must acquire strategic knowledge in some form, since they are able to solve problems that we are confident require strategic knowledge. It seems a reasonable conjecture that this knowledge is often acquired by induction. Texts include example problems that present the steps of solutions in sequence, and teachers solve example problems during class, both before and after students have attempted to solve problems as exercises. The principles of strategic knowledge that must be applied in solving problems probably can be induced as general properties of the sequences of steps that students observe in example solutions. Knowledge that is induced in this way probably is implicit in nature. As with many intellectual skills, when we ask subjects to explain how they decided to perform in the way they did, the answers are not very coherent. Thus, the induced strategic principles appear to be in the form of tacit procedural knowledge involving things the learner is able to do, but not things that the learner can describe or analyze.

It is not surprising that students' knowledge of strategic principles is implicit; it has only been in recent years that our scientific theories have included concepts that make it possible to describe strategic knowledge in explicit ways. In our general wisdom about problem solving, we attribute the skill some students show in problem solving either to their intelligence, or to their motivation in the form of persistence, or at most to their ability to use very general heuristic problem-solving methods. However, when current theoretical

concepts and methods are used to analyze the tasks, the analysis indicates a set of important strategic principles involving planning knowledge that is quite specific to the domain of problems that are analyzed.

A question about instruction arises in a rather obvious way. Now that we have discovered the nature of domain-specific strategic knowledge, should we include it explicitly in the materials of the geometry course? The argument for teaching strategies explicitly is quite straightforward. Strategic knowledge is part of the knowledge that students must acquire in order to solve problems in geometry. It is reasonable to try to teach that knowledge, like other knowledge of the course, in as effective a way as possible. While it is possible that the unguided discovery method that is now used is more effective than a more explicit form of instruction would be, that seems unlikely in the light of the research that has been done on discovery learning. The propositions for inference and concepts for pattern recognition in geometry are taught in the specific form in which they are required for geometry problem solving, and it seems reasonable to treat problem-solving strategies in the same way.

An argument against teaching specific problem-solving strategies explicitly rests on the intuition that with the instructional methods we now use, students are required to actively generate the solutions of problems, and that this is a more valuable learning experience than would be provided if the instructional materials provided step-by-step guidance in methods of solution. The issue is an empirical one, albeit difficult to decide, and it would be desirable to have some empirical comparisons between instructional methods that are based on the two ideas. However, it seems certain that some methods of teaching strategic

principles could be devised that would do more harm than good. It would probably not be helpful to most students to teach about strategies in an abstract way, with the strategic principles divorced from the context of problem solving in which they are used. Successful performance in solving problems probably should be considered as an intellectual skill, and it seems likely that successful instruction in problem-solving strategies will be based on principles of skill acquisition. Since we don't understand very much about the principles of skill acquisition, it is clear that we have a long way to go before we can make definite pronouncements about the relative merits of different forms of instruction in problem-solving strategies. It should be noted, though, that our present methods are quite analogous to the method of teaching swimming that consists of throwing a pupil into the water. That method is successful for some students, but it has obvious negative consequences for others.

Another possibility that I believe should be investigated is inclusion of explicit instruction about problem-solving strategies in the instruction that is given to mathematics teachers. I have not studied geometry teachers' understanding of problem solving in a systematic way, but the teachers with whom I have had conversations have quite an undifferentiated impression of the nature of skill in solving problems. In one meeting of teachers, when I described the strategic component of the problem-solving model Perdix, one teacher responded by asking whether what I was discussing wasn't just the students' intelligence. This teacher's view was that some students are better than others in applying mathematical ideas in problem situations, and that occurs because they are more intelligent. Another teacher proposed



a motivational theory, in which the cause of failure in problem solving arises from a lack of persistence. When difficulties are encountered, some students continue to work on the problem and may eventually find a way to make progress, while others give up as soon as the next move is not obvious. I am sure that both of these views have merit, but they are not the complete story. I am hopeful that teachers might be able to be considerably more helpful in facilitating their students in the acquisition of problem-solving skills if their own understanding of the process became somewhat more sophisticated, with some concepts that refer to various components of the skill rather than being limited to very global concepts of intelligence and persistence.

I will close this discussion of geometry by noting that the cognitive analysis of problem solving has not provided strong recommendations about how to teach the subject matter. It has provided a characterization of the knowledge that a student should acquire, and some of the features of that knowledge raise issues about instruction that appear to be significant and interesting. It may be that specific recommendations about instruction would follow from a cognitive analysis of the learning process itself, but that is a point we will have to look into when we have some theoretical analyses of the learning process.

#### Computation and Understanding in Arithmetic

A second instructional task that we have been studying at Pittsburgh is elementary arithmetic. In this work we have begun with the basics—concepts of addition and subtraction that are taught in the first and second grades. As in the case of geometry, we are attempting to develop a model that represents the knowledge that students acquire if they are

successful in mastering the material they encounter in arithmetic instruction.

Instructional objectives for primary arithmetic have two aspects: skill and understanding. In the domain of skill, students are expected to learn the basic addition and subtraction facts, so they can answer questions such as, "What is  $8 - 3$ ?" or "What is  $3 + 5$ ?" or perhaps " $3 + ? = 8$ ." In the domain of understanding, a variety of tasks are included in the curriculum, and they probably relate to rather different ideas about the nature of understanding. We have focused on the kind of understanding needed for children to be able to apply their knowledge of arithmetic in concrete situations, or in the semi-concrete situations that are presented in the form of word problems.

A substantial number of studies have analyzed processes for answering questions involving basic arithmetic facts. A considerable body of evidence now supports the idea that children use methods based on counting when they answer simple questions such as, " $3 + 5 = ?$ ." The method used by practiced subjects for addition is shown in Figure 1. Evidence supporting this model has been obtained in studies by Groen

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Insert Figure 1 about here

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and Parkman (1972) and by Groen and Resnick (1977). The evidence supports a model of subtraction that is similar in character. If the gap between two numbers in a subtraction problem is small, as in " $8 - 6 = ?$ " the child finds the answer by counting the size of the gap. If the number to be subtracted is small, as in " $8 - 2 = ?$ " the child uses a procedure that requires only a couple of counts—it

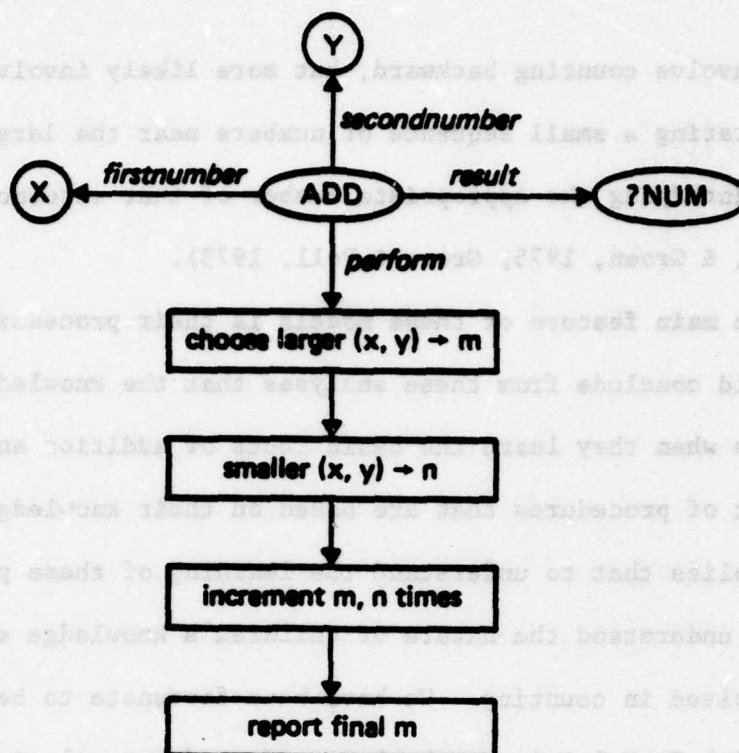


Figure 1. Procedure for answering simple addition questions.



might involve counting backward, but more likely involves some process of generating a small sequence of numbers near the larger term, and then identifying the appropriate member of that sequence (Woods, Resnick, & Groen, 1975; Groen & Poll, 1973).

The main feature of these models is their procedural character. We should conclude from these analyses that the knowledge acquired by students when they learn the basic facts of addition and subtraction is a set of procedures that are based on their knowledge of counting. This implies that to understand the learning of these procedures, we need to understand the nature of children's knowledge structures that are involved in counting. We have been fortunate to be able to collaborate with Rochel Gelman, who has conducted several studies of children's counting, focused on analyzing general principles that children understand and that affect their performance in counting tasks. This collaborative project, in which Mary Riley is also participating, has the goal of representing children's counting knowledge in a simulation model that we test by comparing its performance on various tasks with the performance that Gelman (1978) has reported. A long-term goal is the development of a simulation of learning, in which the knowledge structures that we identify for the counting tasks are transformed into knowledge structures that are capable of performing addition and subtraction.

The second aspect of knowledge about arithmetic involves children's understanding of concepts and procedures. In one test of understanding children are asked to solve problems consisting of brief stories involving quantitative information such as the following: "Jill had three

apples. Betty gave her some more apples. Now Jill has eight apples. How many did Betty give her?"

One project that we have begun is a simulation model of the process of solving arithmetic word problems; in this project I am collaborating with Joan Heller. A model of solving word problems has been developed previously, by Bobrow (1968), but our model is based on quite a different view of the process than Bobrow's was. In Bobrow's model, the main process was translation of the text into a set of simultaneous equations. This process of translation was based as much as possible on syntactic information, and semantic processing occurred only when it could not be avoided. In our model, semantic processing is the main component of the understanding process. The system constructs a semantic network representing the information in the problem. To solve the problem, the system must select an arithmetic operation—for example, addition or subtraction. In our model, the operations are associated directly with structural representations, so there is no intervening process of constructing equations before the operation is chosen.

The processing of a problem by our system is based on a set of schemata that specify alternative structures of quantitative information. The analysis of these schemata has provided the most interesting result of our project thus far. The problems we have analyzed at this point all are solved by addition or subtraction of the numbers given in the problem. We have identified three distinct schemata that we believe are necessary and sufficient for understanding of all the problems that are solved by a single operation of addition or

subtraction. I will refer to these three schemata as Cause/Change, Combination, and Comparison.

The Cause/Change schema is used for understanding situations in which some event changes the value of a quantity. For example, when Betty gives Jill some apples, there is a change in the number of apples that Jill has. The abstract schema that represents such situations is in Figure 2. There are three main components. First, there is an initial quantitative state in which some object  $O$  is associated with some quantity  $P$ . Second, there is some action that in-

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Insert Figure 2 about here

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volves a direction of change, increase or decrease, and an amount  $Q$ , in the object  $O$ . Finally, there is a resulting state in which  $O$  has quantity  $R$ . For example, in the problem where Jill had three apples and got five more from Betty, the object is the set of apples in Jill's possession, the initial amount,  $P$ , is three, the direction of change is increase and the amount of increase,  $Q$ , is five. The question indicates that the final amount,  $R$ , is unknown and the problem is to find that quantity.

Figure 2 indicates that both addition and subtraction are related to the cause/change schema. This is because either operation can be required to solve problems in which the schema is used to represent the information. Consider two kinds of problems in which the unknown quantity is  $R$ , the amount in the final state, with numbers given as the values of  $P$  and  $Q$ . Addition is needed if the direction of the change is an increase, and subtraction is needed if the direction of



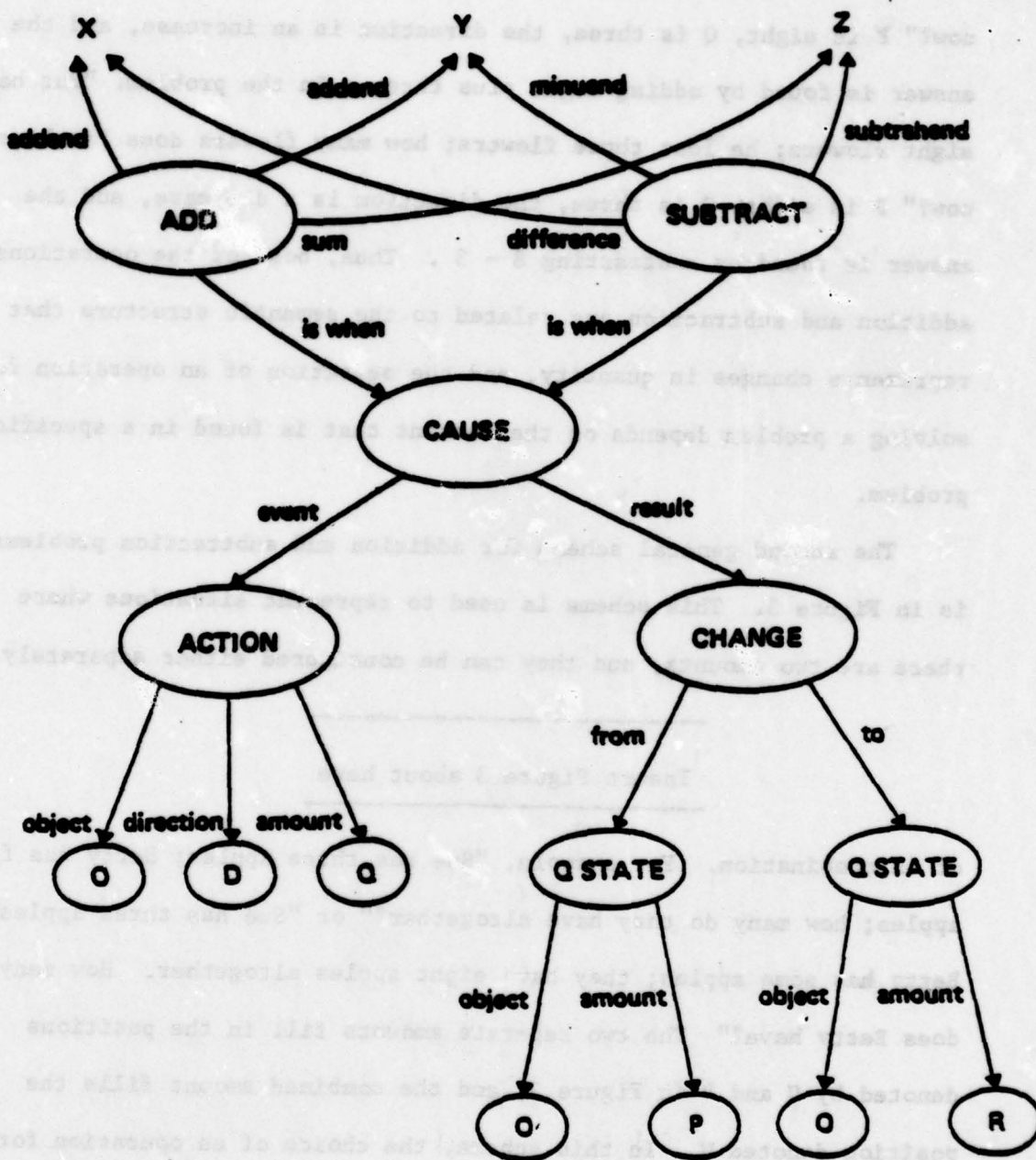


Figure 2. Schema for Representing Problems in Which an Event Causes a Change in a Quantity.

the change is a decrease. For example, in the problem, "Pat had eight flowers; he found three more flowers; how many flowers does Pat have now?" P is eight, Q is three, the direction is an increase, and the answer is found by adding eight plus three. In the problem, "Pat had eight flowers; he lost three flowers; how many flowers does Pat have now?" P is eight, Q is three, the direction is a decrease, and the answer is found by subtracting  $8 - 3$ . Thus, both of the operations addition and subtraction are related to the semantic structure that represents changes in quantity, and the selection of an operation for solving a problem depends on the content that is found in a specific problem.

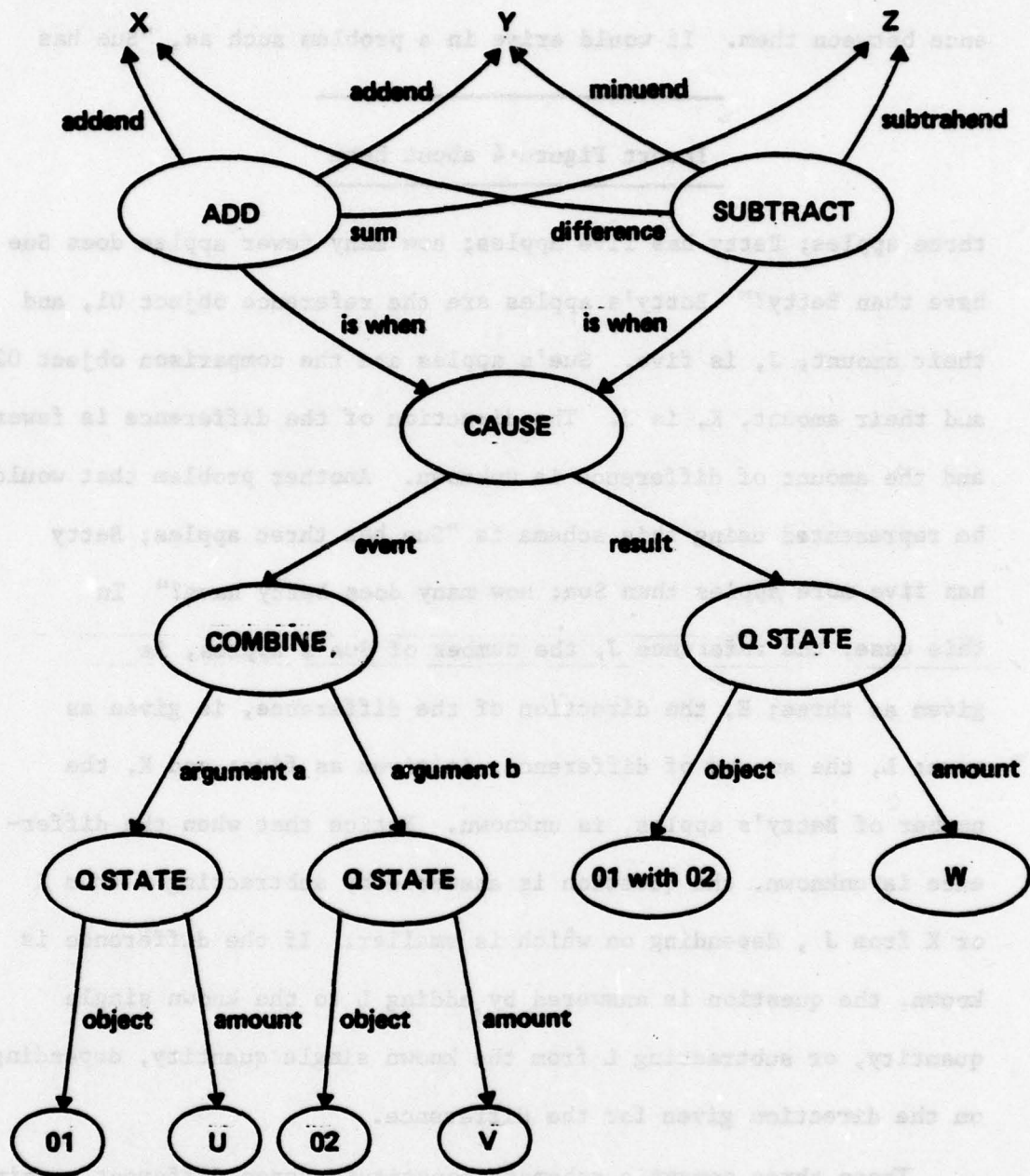
The second general schema for addition and subtraction problems is in Figure 3. This schema is used to represent situations where there are two amounts, and they can be considered either separately

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Insert Figure 3 about here

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or in combination. For example, "Sue has three apples; Betty has five apples; how many do they have altogether?" or "Sue has three apples; Betty has some apples; they have eight apples altogether. How many does Betty have?" The two separate amounts fill in the positions denoted by U and V in Figure 3, and the combined amount fills the position denoted W. In this schema, the choice of an operation for answering a question depends on which of the three quantities is unknown in the question. If the combined amount is unknown, it is found by adding the other two amounts. If one of the separate amounts is unknown, it is found by subtracting the known separate amount from the combined amount.



**Figure 3. Schema for Representing Problems Involving a Combination of Two Quantities.**



The third general schema for addition and subtraction is in Figure 4. This involves two amounts that are compared, and a difference between them. It would arise in a problem such as, "Sue has

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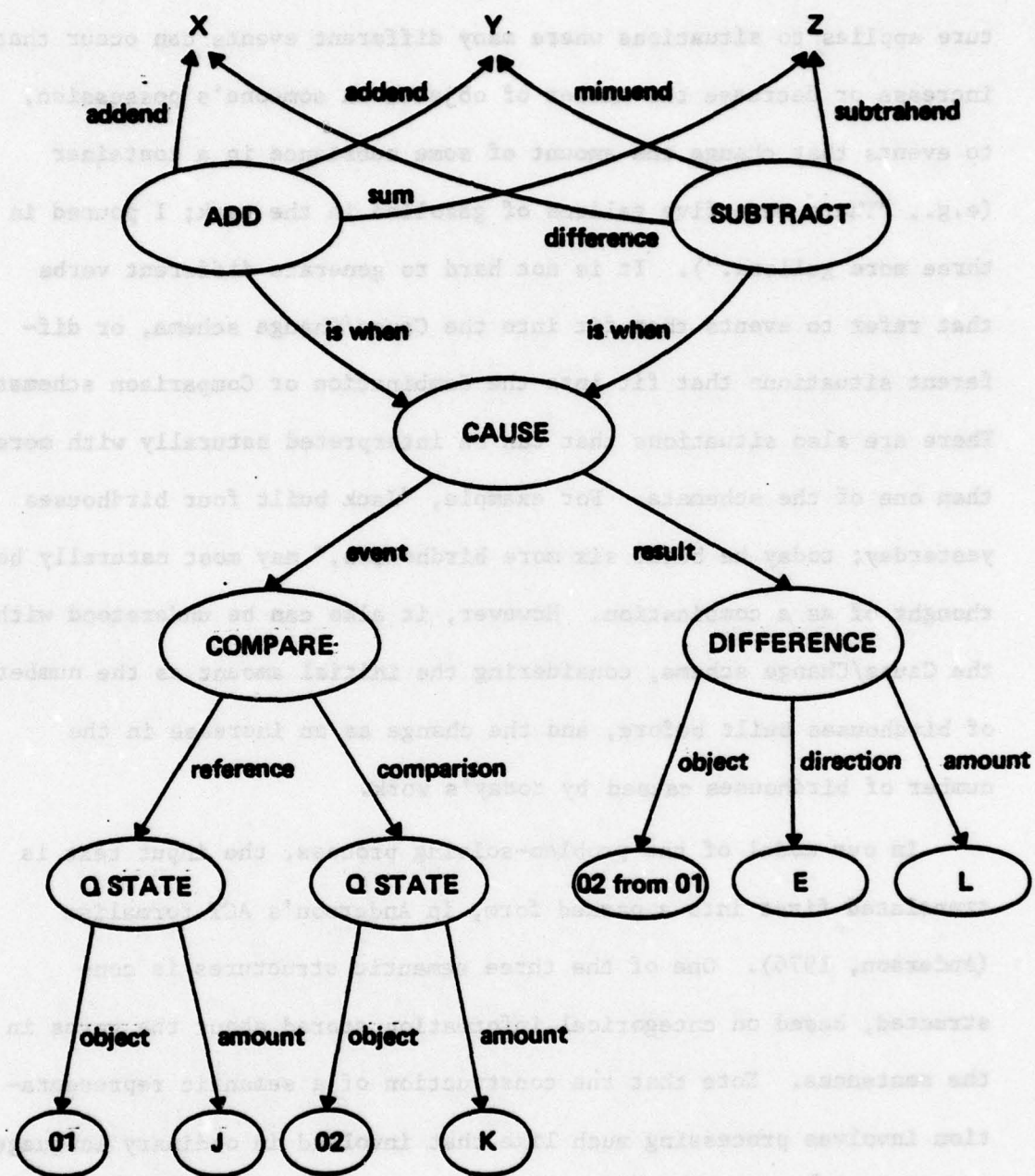
Insert Figure 4 about here

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three apples; Betty has five apples; how many fewer apples does Sue have than Betty?" Betty's apples are the reference object 01, and their amount, J, is five. Sue's apples are the comparison object 02, and their amount, K, is 3. The direction of the difference is fewer, and the amount of difference is unknown. Another problem that would be represented using this schema is "Sue has three apples; Betty has five more apples than Sue; how many does Betty have?" In this case, the reference J, the number of Sue's apples, is

given as three; E, the direction of the difference, is given as more; L, the amount of difference, is given as five; and K, the number of Betty's apples, is unknown. Notice that when the difference is unknown, the question is answered by subtracting J from K or K from J, depending on which is smaller. If the difference is known, the question is answered by adding L to the known single quantity, or subtracting L from the known single quantity, depending on the direction given for the difference.

These three semantic schemata constitute three different meaning structures for addition and subtraction. I think it is appropriate to say that these arithmetic concepts are ambiguous. They have distinct and incompatible meanings. On the other hand, addition and subtraction are genuine abstractions in relation to the Cause/Change,



**Figure 4. Schema for Representing Problems Involving a Comparison of Two Quantities.**

Combination, and Comparison meaning structures. These, in turn, are relatively abstract themselves. For example, the Cause/Change structure applies to situations where many different events can occur that increase or decrease the number of objects in someone's possession, to events that change the amount of some substance in a container (e.g., "There were five gallons of gasoline in the tank; I poured in three more gallons."). It is not hard to generate different verbs that refer to events that fit into the Cause/Change schema, or different situations that fit into the Combination or Comparison schemata. There are also situations that can be interpreted naturally with more than one of the schemata. For example, "Jack built four birdhouses yesterday; today he built six more birdhouses," may most naturally be thought of as a combination. However, it also can be understood with the Cause/Change schema, considering the initial amount as the number of birdhouses built before, and the change as an increase in the number of birdhouses caused by today's work.

In our model of the problem-solving process, the input text is translated first into a parsed form, in Anderson's ACT formalism (Anderson, 1976). One of the three semantic structures is constructed, based on categorical information stored about the verbs in the sentences. Note that the construction of a semantic representation involves processing much like that involved in ordinary language processing, with inferences made in order to achieve a coherent structure. However, the inferences made in the context of arithmetic word problems are quite different from those made in other contexts, such as ordinary stories. If the sentence, "Betty gave Jill five apples" were encountered in a story, the reader would probably be making



inferences about Betty and Jill's friendship, or some general goal of Betty's such as a hope that Jill would reciprocate by sharing something that Betty wants (cf. Schank & Abelson, 1977). In the context of an arithmetic problem, if a person already has the information that Jill had three apples before, then the sentence, "Betty gave Jill five apples" produces the inference that a change occurred in the number of Jill's apples, the direction of the change was an increase, and the amount of the change was five.

When a semantic representation has been constructed, the answer is obtained by applying an arithmetic operation. The first three columns in Table 1 specify 14 different structures that can result from representing different addition and subtraction problems. One possible

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Insert Table 1 about here

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theory is that each of these is simply associated with one of the operations, along with a procedure for assigning the quantities in the problems as arguments of the procedures. The form of the model that we have programmed is based on a somewhat different intuition which we consider plausible, but not firm. The current model has direct associations from six of the semantic structures to operations. For the remaining structures, a transformation is required to obtain a representation that is associated with one of the operations. For example, for a problem such as, "Jill had three apples; Betty gave her some more apples; now Jill has eight apples; how many apples did Betty give her?" the model first generates a Cause/Change structure with three as the starting quantity, eight as the final quantity, an

Table 1

## Selection of Arithmetic Operators

<u>Schema</u>	<u>Direction</u>	<u>Unknown</u>	<u>Decision</u>
Cause/Change(P,Q,R)	Increase	Result, R	Addition ( $P + Q$ )
Cause/Change(P,Q,R)	Decrease	Result, R	Subtraction $P - Q$
Cause/Change(P,Q,R)	Increase	Change, Q	Transform to Combine(P,Q,R)
Cause/Change(P,Q,R)	Decrease	Change, Q	Transform to Combine(R,Q,P)
Cause/Change(P,Q,R)	Increase	Start, P	Transform to Combine(P,Q,R)
Cause/Change(P,Q,R)	Decrease	Start, P	Transform to Combine(R,Q,P)
Combine(U,V,W)	———	Combined Amount, W	Addition ( $U + V$ )
Combine(U,V,W)	———	Separate Amount, V	Subtraction ( $W - U$ )
Compare(J,K,L)	More	Difference, L	Subtraction ( $K - J$ )
Compare (J,K,L)	Fewer	Difference, L	Subtraction ( $J - K$ )
Compare(J,K,L)	More	Second Amount, K	Transform to Combine(J,L,K)
Compare (J,K,L)	Fewer	Second Amount, K	Transform to Combine(K,L,J)
Compare (J,K,L)	More	First Amount, J	Transform to Combine(J,L,K)
Compare(J,K,L)	Fewer	First Amount, J	Transform to Combine(K,L,J)

increase as the direction, and the amount of increase unknown. This is the structure described on the third line of Table 1. Then this structure is transformed to a Combine structure, with three as the first separate amount, eight as the combined amount, and the second separate amount unknown. This is the structure shown on line eight of Table 1. This new structure is associated with the operation of subtraction, so the system then chooses that operation.

The choice of Combine as the canonical structure for missing addend problems is largely speculative on our part, though there is some suggestive evidence in Case's (1978) work that is consistent with our intuition. We consider the specific set of decision rules in Table 1 to be quite arbitrary, and probably different individuals have different decision rules associated with the semantic structures. The nature of these decision and transformation processes remains an open question in our research, and Table 1 should be considered as illustrative of the kinds of procedures that seem plausible in the framework that we are using.

The idea of a system that solves word problems without generating equations is encouraged by the fact that children can solve many word problems before they have any knowledge of equations. In fact, there are data showing that children can solve some word problems before they begin to learn arithmetic at all (Buckingham & MacLachy, 1930). The supply of data about solution of word problems by young children is not large, perhaps because it is much more convenient to present word problems as test items to children who are able to read the problems from written text. One of our current projects involves collecting some systematic data to identify the abilities of young



children to understand the kinds of information involved in simple word problems.

In one experiment conducted by Mary Riley, second-grade children were asked to solve a series of word problems that were designed to provide information about relative difficulty of the three semantic structures that we identified in the theoretical analysis described above. A sample of the problems used in the experiment are shown in Table 2. In the experiment, students were asked to solve the problems, and were also asked to represent the problems using sets of blocks. Table 3 shows the structural descriptions of the nine kinds of problems used, and also shows the proportions of correct answers and the proportions of correct representations that the children produced with blocks.

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Insert Tables 2 and 3 about here

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The main finding is that the semantic schemata involved in problems were rather strong determiners of problem difficulty for these children. They had little difficulty with any of the problems with the Cause/Change structure. While the Combination problems with the combined amount unknown were all solved correctly, the students were not as successful with the Combination problems with one of the separate amounts unknown. This finding casts doubt on the assumption about decision rules shown in Table 1 that missing addend problems are all transformed into Combination structures. We are collecting further data on this matter, but if results like those in Table 3 are typical, we should revise our assumptions about the nature of

Table 2

## Examples of Problems

<u>Schema</u>	<u>Example</u>
Cause/Change	<ol style="list-style-type: none"> <li>1. Joe has 3 marbles. Tom gives him 5 more marbles. How many marbles does Joe have now?</li> <li>2. Joe has 8 marbles. He gives 5 marbles to Tom. How many marbles does Joe have now?</li> <li>3. Joe has 3 marbles. Tom gives him some more marbles. Now Joe has 8 marbles. How many marbles did Tom give him?</li> <li>4. Joe has 8 marbles. He gives some marbles to Tom. Now Joe has 3 marbles. How many marbles did he give to Tom?</li> </ol>
Combination	<ol style="list-style-type: none"> <li>5. Joe has 3 marbles. Tom has 5 marbles. How many marbles do they have altogether?</li> <li>6. Joe and Tom have 8 marbles altogether. Joe has 3 marbles. How many marbles does Tom have?</li> </ol>
Comparison	<ol style="list-style-type: none"> <li>7. Joe has 3 marbles. Tom has 5 more marbles than Joe. How many marbles does Tom have?</li> <li>8. Joe has 8 marbles. He has 5 more marbles than Tom. How many marbles does Tom have?</li> <li>9. Joe has 5 marbles. Tom has 8 marbles. How many more marbles than Joe does Tom have?</li> </ol>

Table 3

**Problem Structures and Proportions of Correct Problem  
Answers and Representations**

<u>Problem Type</u>	<u>Schema</u>	<u>Direction</u>	<u>Unknown</u>	<u>Correct Answers</u>	<u>Representations</u>
1.	Cause/Change	Increase	Result	1.00	1.00
2.	Cause/Change	Decrease	Result	1.00	1.00
3.	Cause/Change	Increase	Change	.83	.94
4.	Cause/Change	Decrease	Change	1.00	1.00
5.	Combine	—	Combined Amount	1.00	1.00
6.	Combine	—	Separate Amount	.67	.77
7.	Compare	More	Comparison Amount	.56	.28
8.	Compare	More	Reference Amount	.28	.50
9.	Compare	More	Difference	.42	.83



transformations that are typically performed with Cause/Change and Combination problems.

The most striking finding of this experiment is that all of the problems that have Comparison structures were relatively difficult for these second-grade children. One interesting item was the discrepancy between the proportions of correct answers and correct representations in problems of Type 7, relative to problem Types 8 and 9. The higher proportion of correct answers for Type 7 apparently was due to a tendency for students to add the numbers in the problem, whether or not they understood the problems. When students were asked to show the relationships using blocks, these were the hardest problems of the set used. In the two remaining types of problems with Comparison structures, representation using blocks was more successful than problem solution, perhaps because the blocks provided a method of holding the quantitative information in external memory.

The analysis of semantic processing in solution of word problems provides an interesting suggestion regarding instruction. If we are correct, the process of solving a word problem often involves construction of a semantic representation that is only indirectly related to the operations of addition and subtraction that are used to solve the problems, but that is nonetheless an important component of the process. The suggestion that this hypothesis leads to is that students might be instructed to identify the various general semantic structures that occur word problems, and relate them to arithmetic operations in appropriate ways. In arithmetic, this would involve training in representing problem situations as one of the three general schemata: change in a quantity, a combination or a comparison; and teaching them the connections

between those representations and the addition and subtraction operations. One approach that seems worth trying would be to use techniques of the kind used in concept formation tasks to train students to attend to the relevant dimensions of information. Many of the training procedures used in experiments that have been concerned with training children to perform more successfully on Piagetian tests of cognitive development can be interpreted as concept-formation procedures in which children learn to attend to the features of the situation that are relevant for the task. Gelman's (1969) study of training for number conservation is an important example in which the discrimination-learning paradigm was adopted explicitly.

A second issue that arises involves the way in which computational skill is acquired. I have already discussed the fact that at the beginning of instruction basic arithmetic, children have relatively sophisticated knowledge about counting, and that this is almost certainly an important knowledge base for their acquisition of basic arithmetic facts of addition and subtraction. An additional issue involves children's understanding of these facts. The instructional materials used in primary grades emphasize use of manipulative materials, such as blocks or plastic counters, in providing alternative representations of addition and subtraction facts. The idea that seems to underlie this instruction is that students will be able to understand the operations performed with blocks and other concrete manipulative materials relatively easily, and these will provide a cognitive basis for their understanding of arithmetic expressed in symbolic notation.

When we began our study of primary arithmetic in 1976, we planned to focus our attention on relationships between formal notation of

arithmetic and manipulations of concrete materials such as blocks, plastic counters, and the number line. Our initial exploratory work using these materials was surprisingly discouraging. Rather than understanding operations on manipulative materials easily, children seemed to have considerable difficulty. The number line was especially troublesome as a medium for representing quantitative information, and we were informed that the children had not received much instruction involving the number line. We were led to wonder whether the children's general understanding of operations with concrete materials may depend rather strongly on the instruction they have received, rather than being something they comprehend easily and naturally. We have not pursued this issue in detail; however, the experience of our informal explorations was sufficiently discouraging that we moved our research program in another direction.

The direction in which we have developed our research is the study of processes of solving word problems, as I have described in this paper. Children seem to have considerable ability to understand information that describes relationships among quantities in concrete situations involving changes in possession, location of objects, and so on. Our current conjecture is that children's ability to understand and solve word problems might be exploited much more than it is in present instructional practice as a part of the cognitive basis for the acquisition of arithmetic concepts and operations. Rather than basing instruction on relatively abstract representations such as blocks or the number line, we wonder whether addition and subtraction (and later, the more advanced topics of arithmetic) might be taught in relation to more concrete events and situations where people give things



to each other, more objects from one room to another, and so on. This involves viewing problem solving as a basis for instruction arithmetic, rather than as a skill that is more complex than arithmetic knowledge and that has to be built on top of the more basic knowledge of computation. The issue has ramifications that implicate fundamental aspects of the current structure of our teaching of mathematics in the schools, and we have only begun to touch the edges of some of these. However, the ideas seem plausible, and we look forward to a lively period of exploration and research in the years ahead.

### Conclusions

In my concluding comments, I will try to extrapolate from the kinds of results we have obtained in our studies of geometry and primary arithmetic. The kinds of issues that are raised by those findings arise in other domains as well, and it seems a reasonable conjecture that there are possibilities for exploring alternative methods of instruction in a number of different domains that correspond to the possibilities that I have been suggesting in the domain of mathematics.

First, the issue of teaching problem solving strategies in geometry seems quite clearly applicable in other domains where students are trained in problem solving. Strategic knowledge in a problem-solving domain consists of knowledge of the kinds of subgoals that are useful in various problem situations and the plans that are helpful in achieving various goals and subgoals. One advantage of teaching that knowledge to a student in explicit form is that the student will then have a better understanding of her or his own problem solving achievements (cf. Brown, Collins, & Harris, 1977). It would be reasonable to expect that this might facilitate transfer to other problem-solving tasks,

although this conjecture remains to be tested. Explicit instruction in a problem-solving domain could have considerable facilitating effects on students' abilities to solve problems within the domain of instruction, but there may be potential hazards of making strategic knowledge too explicit, if it reduced the educational benefits that at least some learners now receive by finding their own solutions for problems. It seems quite likely, however, that if a more detailed analysis of strategic knowledge in a problem domain were taught to individuals who are instructors in that domain these individuals would have a better understanding of what their students are required to learn in order to succeed as problem solvers and could interact with their students more effectively in instructional situations.

The second general issue raised by the analyses I have presented is the issue of teaching students how to represent problem situations. There is a very large experimental literature on the process of learning the relevant attributes of a categorical concept, and an interesting extension to that literature has been given in Winston's (1975) analysis of acquisition of concepts in the blocks world. The general idea of analyzing the relevant features of problem domains and then giving specific training in identifying those features seems to be widely applicable. Recent studies by Larkin (1977) and by Simon and Simon (1978) have indicated that a major difference between expert and novice problem solvers in physics arises from the expert's construction of an abstract representation of the problem situation, in contrast to the novice's more direct attack on the problem. One interpretation of the result is that by achieving a coherent representation of the situation, the expert avoids the need for extensive

problem-solving search because the expert's representation contains information needed to select appropriate problem-solving operators directly. The well known studies of expert chess and Go players' to encode complex game positions rapidly (Chase & Simon, 1973; Reitman, 1976) attest further to the importance of knowledge for representing problem situations to successful problem solving performance.

While the experimental literature on concept formation provides a useful starting point for a program of developing instructional technology for representational knowledge, we probably will encounter some important differences when we study concept formation in the domain of problem representation. Traditional study of concept formation emphasized features that permitted classification of stimuli, and used simple perceptual features as much as possible. In the representation of problem situations, the important thing is to find features that are relevant to the selection of a problem-solving method, rather than features that simply distinguish one category of situations from another. This means that the concepts to be acquired are components of a decision process, rather than simple labels. Further, the powerful representations that experts construct apparently involve complex and abstract relationships in the problem situation, rather than simple perceptual attributes. We will need to extend our technology for teaching concepts considerably in the domain of problem solving representations, but it seems a promising and generally applicable idea.

The third general issue raised by these analyses involves the acquisition of procedural knowledge in meaningful ways. It has always seemed reasonable to teach procedures in contexts that involved the



situations in which the procedures were to be used to solve problems, both because that should make it more likely that the learner would be able to apply the knowledge appropriately and because in that way the new procedures would be more meaningful. However, the analysis of arithmetic problems and procedures may illustrate some of the reasons why that old truism is correct. The problem solving contexts in which procedures are applied may indicate important semantic distinctions that should be considered as differences in meaning of the procedural concepts that are involved in the instruction. These distinctions are probably important for students to understand, since they are relevant components of the situations in which they are expected to use procedures to solve problems. They also may be important mediating concepts that are needed to provide understanding of the nature of relationships between concrete problem situations and the abstract ideas involved in problem-solving methods.

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